FUNDAMENTALS OF PHYSICS

By

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Introduction

Physics is the science of matter and its motion as well as space and time.

It uses concepts such as energy, force, mass, and charge. Physics is an experimental science, creating theories that are tested against observations.

Broadly, it is the general scientific analysis of nature, with a goal of understanding how the universe behaves. Physics is an empirical study. Also physics is a science of measurement.

Physical systems

- Everything that can be analyzed by the physical law is a physical system. There are numbers of physical systems. We will study here mechanical system.

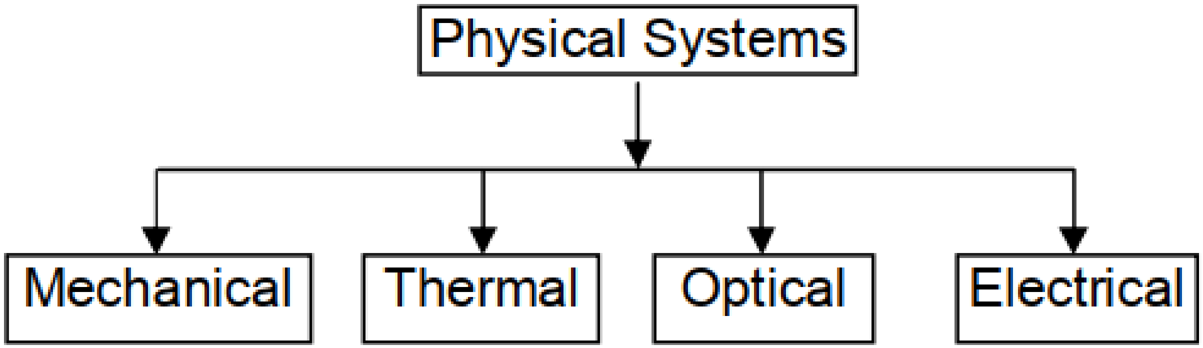

Figure 1: Some common physical systems

Measurement : Measurement is the process of estimating the magnitude of some attribute of an object, such as its length or weight, relative to some standard (unit of measurement), such as a meter or a kilogram.

- **Physical quantity:** Any number or sets of number used for a quantitative description of a physical phenomenon is called a physical quantity.

- **Unit:** A unit of measurement is a standardized quantity of a physical property, used as a factor to express occurring quantities of that property. For example, suppose a rod is 30*m* long, i.e. it is 30 times long as an object whose length has been defined to be one meter. Such a standard is called a unit of the quantity. There are different systems of unit in the world.

- **1. MKS System:** Unit of length- meter Unit of mass- kilogram Unit of time- second
- **2. CGS system:** Unit of length- centimeter Unit of mass- gram Unit of time- second
- **3. FPS system:** Unit of length- foot

UNIT CONSISTENCY

Relations among physical quantities are often expressed by equations in which the quantities are represented by algebraic symbols, only if they have the same units.

An equation must always be dimensionally consistent; this means that two terms may be added or equated only if they have the same units.

Suppose, if a body moving with constant speed v travels a distance s in time t, these quantities are related by the equations $s = v * t$.

If s is measured in meters, then the product must also have units of meters. Thus an equation must always be dimensionally consistent. Physical quantities are often divided into

Fundamental quantities and **derived quantities**.

FUNDAMENTAL QUANTITIES:

- Quantities are organized in a dimensional system built upon base quantities, each of which is regarded as having its own dimension. Fundamental quantities are not defined in terms of other quantities.
- For example, mass, length, time, etc. According to the international System of Quantities (ISQ) the fundamental quantities and their dimensions are listed in the following table:

ISQ FUNDAMENTALS QUANTITIES

Name	Symbol for quantit y	Symbol for dimensi on	SI fundamental unit
Length	l	L	meter
Time	t	T	second
Mass	m	M	kilogram
Electric current	I	I	ampere
Thermodyna mic temperature	T	q	Kelvin
Amount of substance	n	N	mole
Luminous intensity	Iv	J	candela

SI DERIVED UNITS

Other quantities, called *derived quantities*, are defined in terms of the seven base quantities via a system of quantity equations. The *SI derived units* for these derived quantities are obtained from these equations and the seven SI base units. Examples of such SI derived units are given in Table below,

Derived quantity	Name	Symbol
area	square meter	m2
volume	cubic meter	m3
speed, velocity	meter per second	m/s
acceleration	meter per second squared	m/s2
wave number	reciprocal meter	m-1
mass density	kilogram per cubic meter	kg/m3
specific volume	cubic meter per kilogram	m3/kg
current density	ampere per square meter	A/m2
magnetic field strength	ampere per meter	A/m
mount-of-substance conc.	mole per cubic meter	mol/m3

The End

The motion of bodies and the application of Newton's laws.

The relation ship between work and energy

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Force

It is the force that enables us to do any work.
To do anything, either we pull or push the object. Therefore, pull or push is called force.

Example: to open a door, either we push or pull it. A drawer is pulled to open and pushed to close.

Effect of Force

- ***Force can make a stationary body in motion***. For example a football can be set to move by kicking it, i.e. by applying a force.

- ***Force can stop a moving body***: For example by applying brakes, a running cycle or a running vehicle can be stopped.

  - ***Force can change the direction of a moving object***. For example; By applying force, i.e. by moving handle the direction of a running bicycle can be changed. Similarly by moving steering the direction of a running vehicle is changed.

- ***Force can change the speed of a moving body***: By accelerating, the speed of a running vehicle can be increased or by applying brakes the speed of a running vehicle can be decreased.

- ***Force can change the shape and size of an object***. For example: By hammering, a block of metal can be turned into a thin sheet. By hammering a stone can be broken into pieces.

Types of Force:

1. Balanced Forces
2. Unbalanced Forces

Balanced Forces

If the resultant of applied forces is equal to zero, it is called balanced forces.
Example: In the tug of war if both the teams apply similar magnitude of forces in opposite directions, rope does not move in either side. This happens because of balanced forces in which resultant of applied forces become zero.

Balanced forces do not cause any change of state of an object. Balanced forces are equal in magnitude and opposite in direction.
Balanced forces can change the shape and size of an object. For example - When forces are applied from both sides over a balloon, the size and shape of balloon is changed.

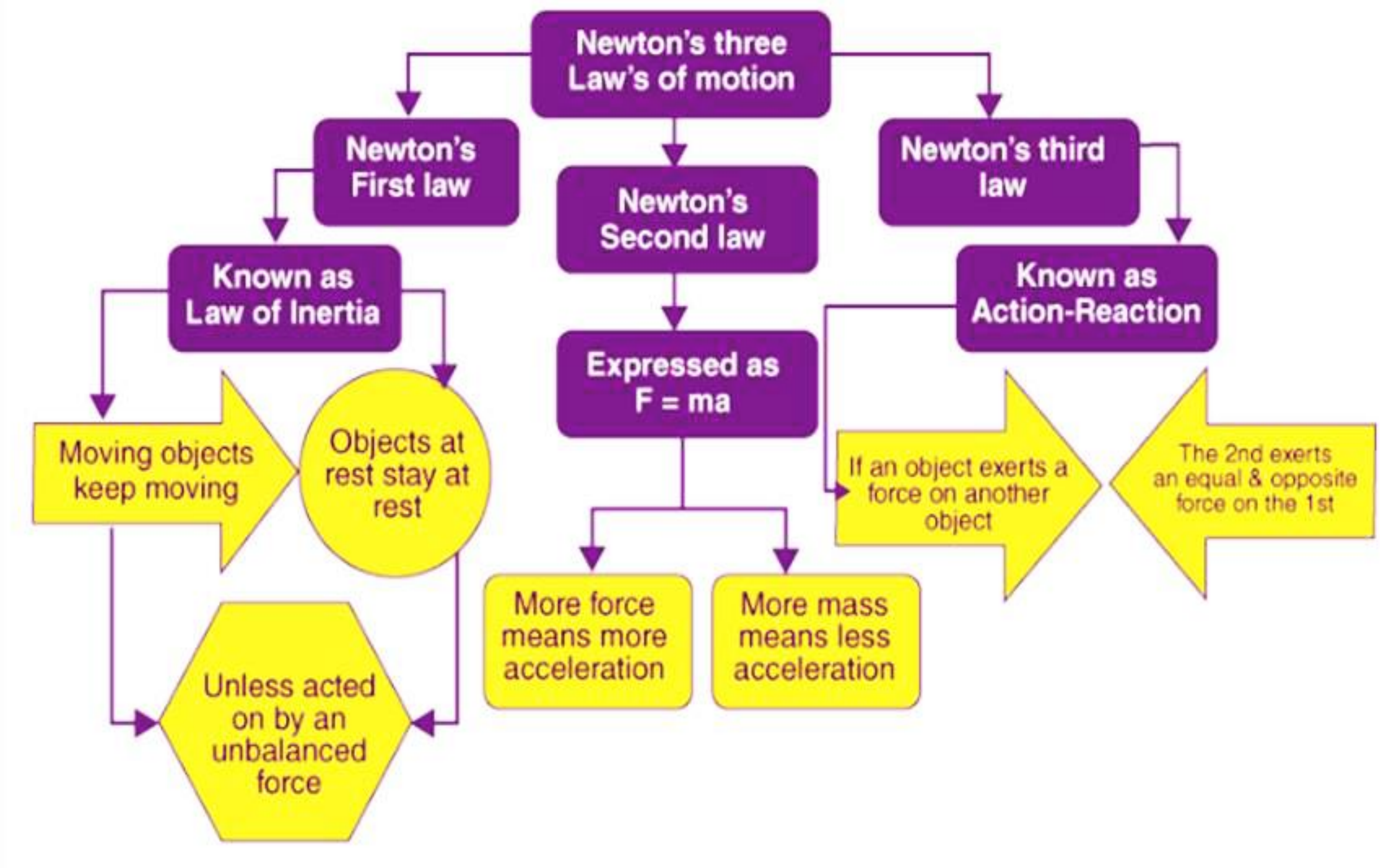
Unbalanced Forces

If the resultant of applied forces are greater than zero the forces are called unbalanced forces. An object in rest can be moved because of applying unbalanced forces.

Unbalanced forces can do the following:

- Move a stationary object.
- Increase the speed of a moving object.
- Decrease the speed of a moving object.
- Stop a moving object.
- Change the shape and size of an object

Newton's laws of motion-Momentum-Work & Energy



Newton’s laws of motion

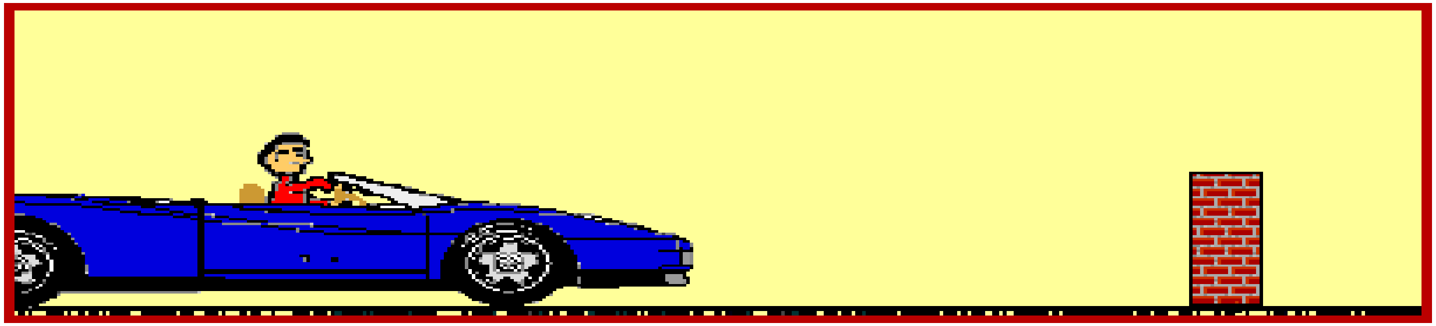
three statements describing the relations between the forces acting on a body and the motion of the body, first formulated by English physicist and mathematician **Isaac Newton,** which are the foundation of classical mechanics.

Newton’s First Law

- An object at rest tends to stay at rest and an ob[illegible]n tends to stay in motion unless acted upon by a [illegible] force.

What does this mean?

- An object at rest remains at rest as long as no **net force** acts on it.
- An object moving with **constant velocity** continues to move with the same speed and in the same direction (the same velocity) as long as no net force acts on it.
- Keep on doing what it is doing .

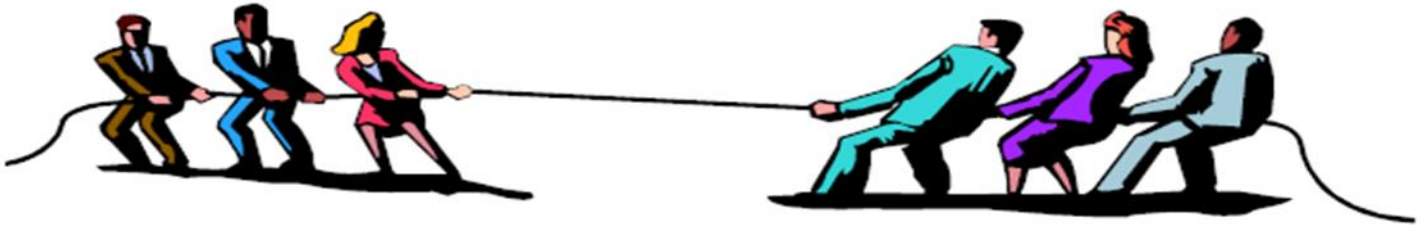


Some Examples from Real Life

- A soccer ball is sitting at rest. It takes an unbalanced force of a kick to change its motion.

Example 2

Two teams are playing tug of war. They are both exerting equal force on the rope in opposite directions. This balanced force results in no change of motion.



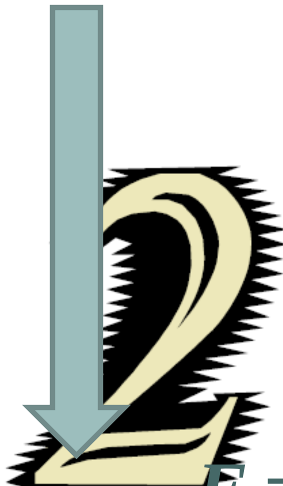
- Newton’s First Law is also called the *Law of Inertia*

- Inertia: the tendency of an object to resist changes in its state of motion.

- The First law states that all objects have inertia. The more mass an object has, the more inertia it has (and the harder it is to change its motion).

Newton's Second Law

Force equals mass times acceleration.



$$F = ma$$

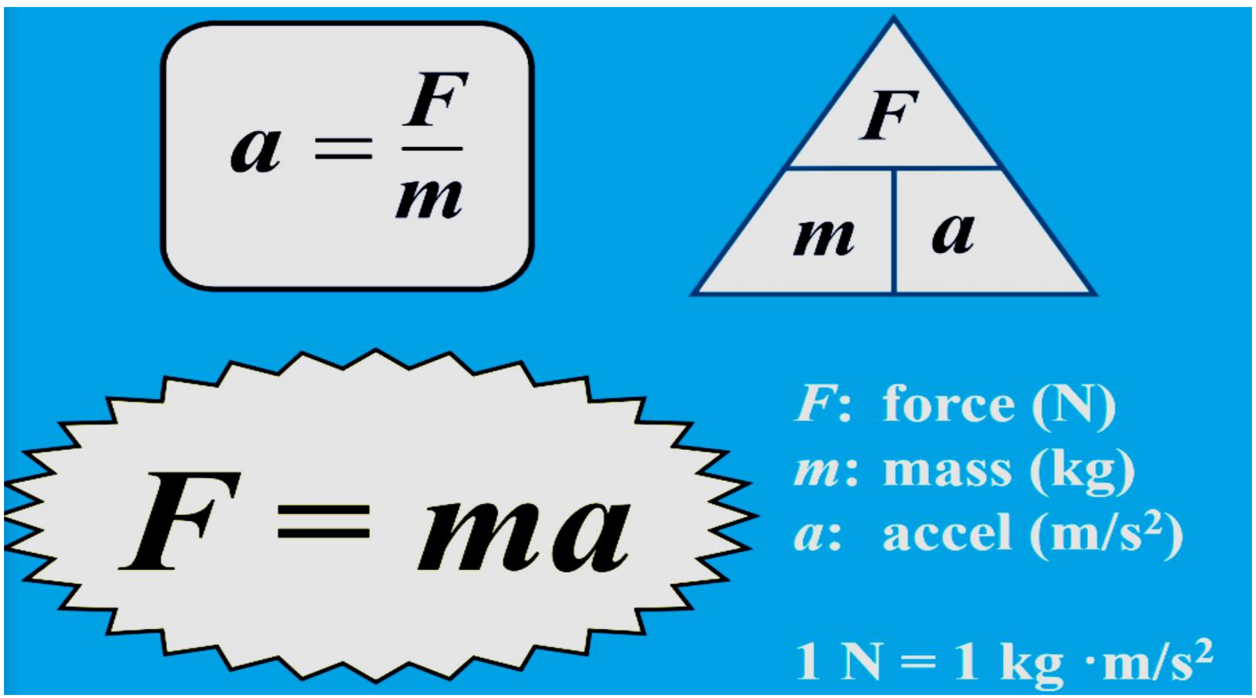
Acceleration: a measurement of how quickly an object is changing speed.

Gravity : force of attraction between any two objects in the universe

increases

mass increases

distance decreases



Example 1 What force would be required to accelerate a 40 kg mass by 4 m/s^2?

F=?
m=40 Kg
a=4 m/s^2

Solution

F=ma
F=(40 Kg)(4 m/s^2)
F=160 N

Example 2 A 4.0 kg shot put is thrown with 30 N of force. What is its acceleration?

F=30
m=4 Kg a=?

Solution

F=ma
30 N=(4 Kg)(a)
a=7.5 m/s^2

Example 3 The frog leaps from its resting position at the lake's bank onto a lily pad. If the frog has a mass of 0.5 kg and the acceleration of the leap is 3 m/s^2, what is the force the frog exerts on the lake's bank when leaping?

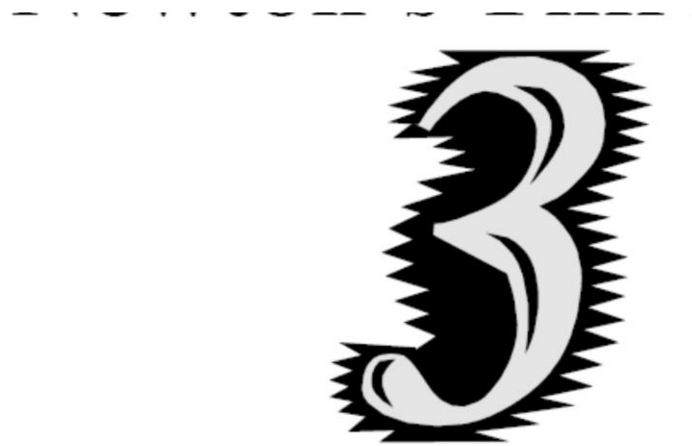
(A) 0.2 N
(B) 0.8 N
(C) 1.5 N
(D) 6.0 N

Formula chart says F=ma, m is mass in kg, a is acceleration in m/$\boldsymbol{s^2}$.
So, 0.5 kg x 3 m/s^2= 1.5 N

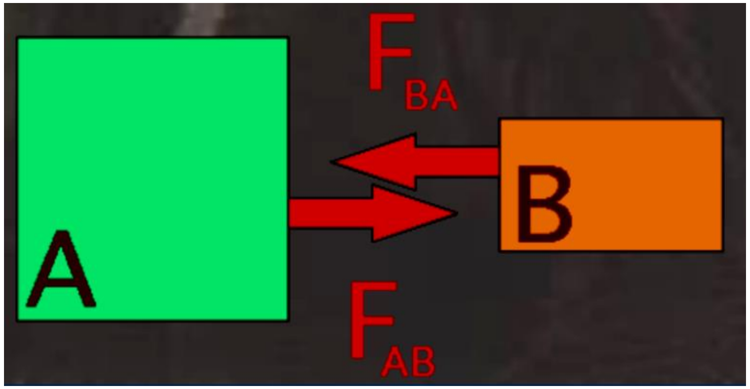
More about F = ma
If you double the mass, you double the force. If you double the acceleration, you double the force.

What if you double the mass and the acceleration?
(2m)(2a) = 4F
Doubling the mass and the acceleration quadruples the force.
So . . . what if you decrease the mass by half? How much force would the object have now?

Newton's Third Law



For every action there is an equal and opposite reaction.



What does this mean?

For every force acting on an object, there is an equal force acting in the opposite direction. Right now, gravity is pulling you down in your seat, but Newton's Third Law says your seat is pushing up against you with equal force. This is why you are not moving .There is a balanced force acting on you–gravity pulling down, your seat pushing up.

Momentum

The product of an object's mass and its speed. A force applied to an object causes a change in its momentum.

p(momentum)= m(mass) x v(velocity)

$$\boldsymbol{p = mv}$$

common unit for momentum **(kg x m/s)**

Example 4 A ball moving at 30 m/s has a momentum of 15 kg.m/s The mass of the ball is —

A. 45 kg
B. 15 kg
C. 2.0 kg
D. 0.5 kg
Formula Page says that Momentum = Mass x Velocity
So, 15 kg.m/s = m (30 m/s) solving for **m** it is:

Example 5 The momentum of a second bumper car is 675 kg.m/s What is its velocity if its total mass is 300 kg?

P=675 kg.m/s
m=300 Kg
v=?
p = mv
675 kg.m/s =(300 Kg)(*v*)
***v*=2.25 m/s**

Work

Work: application of a force to an object that results in the movement of the object over a certain distance.

W = F x d

The work done by forces on an object = changes in energy for that object.

Work & Energy UNITS: Force (F) = Newton, Distance (d) = Meters

Work = N x m or Joule (J)

Example 6

How much work is performed when a50kg crate is pushed 15m with a force of 20N?

A. 300 J

B. 750 J

C. 1,000 J

D. 15,000 J

Use the formula Work = Force x distance (W=F x d)

Force of 20 N x 15 meters = 300 Joules Answer

Example 7 If a force of 100 newton's was exerted on an object and no work was done, the object must have ……..

A. accelerated rapidly
B. remained motionless
C. decreased its velocity
D. gained momentum

Work = Force x Distance (W=F x d)

Work = 0 Force = 100 N so

0 J = 100 N x d
distance must be 0

It did not move!

Work & Energy

Work = Δ Energy

F x d = Δ E (Energy)

This is known as Mechanical Energy

Mechanical Energy is energy of position or energy of movement.

Energy of position is Potential (PE) = weight * height

PE = mgh

Energy of movement is Kinetic (KE) = ½ mass * speed2

KE = ½ m x v2

Example 8

If your backpack weighs 100N and you carry it 8m up a flight of stairs. How much work have you done?

Work = 100N x 8m = 800 joules (J)

How much gravitational potential energy has the backpack gained?

PE = mg h = 10kg x $10m/s^2$

8m = 800 joules (J)

The End

electric charge, electric field, electric charge and structure of matter

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Electric Charge and the Structure of Matter

- The structure of atoms can be described in terms of three particles

The negatively charged **electron**

- $\boldsymbol{Mass = 9.109 \times 10-31kg}$

The positively charged **proton**

- $\boldsymbol{Mass = 1.673 \times 10-27kg}$

The uncharged **neutron**

- $\boldsymbol{Mass = 1.675 \times 10-27kg}$

Structure of atoms

- ***Charge Carried by Electrons and Protons***

- A model of an atom with negative electrons orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons.** Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral,carrying no charge.)The charges of electrons and protons are identical in magnitude but opposite in sign. The magnitude of this basic charge is

$$\boxed{q = 1.6 \times 10^{-19}\ Coulomb\ (C)}$$

❖ ***Conductors and insulators***

Materials that allow easy passage of charges are called conductors. (e.g. most metals)
Materials that resist electronic flow are called insulators. (e.g. glass, wood).

Coulomb's Law

The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

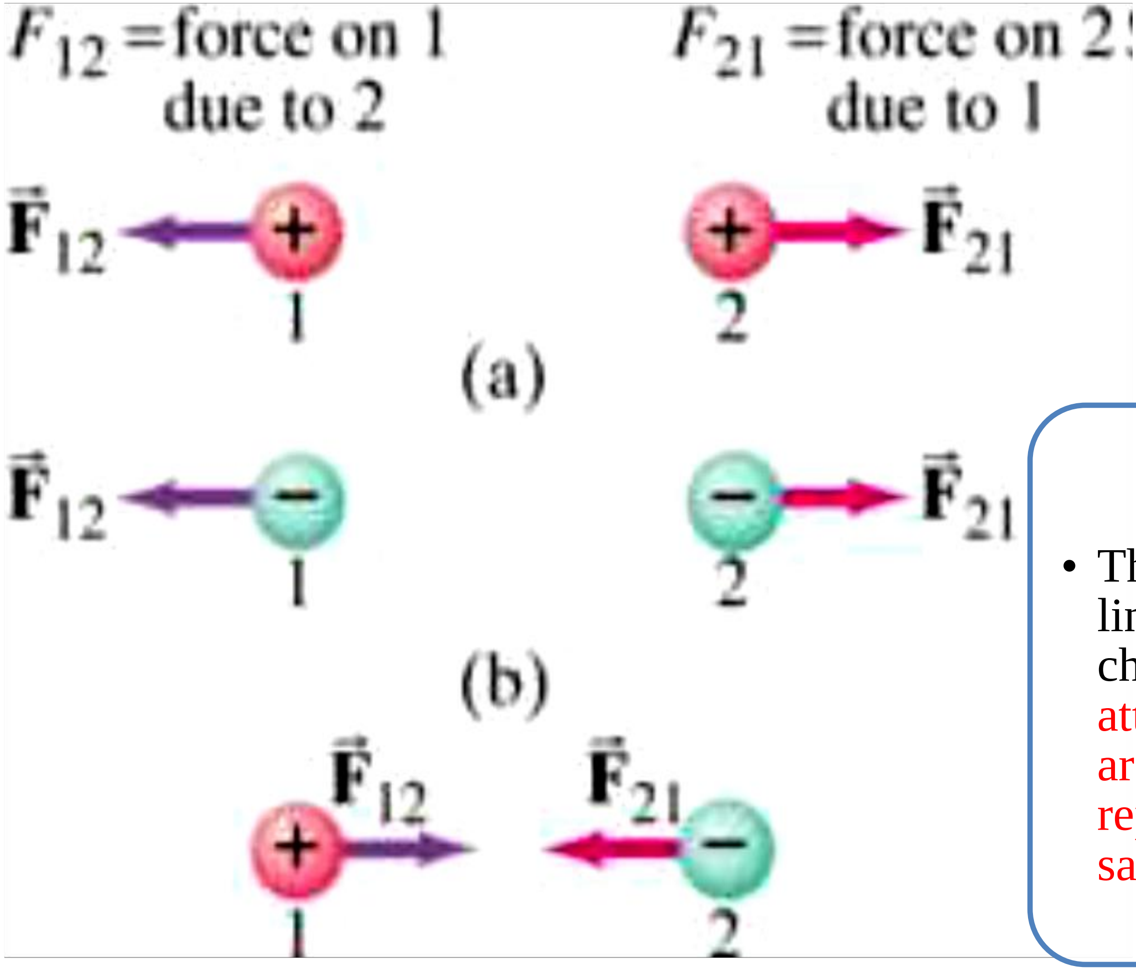
Units: q_1 and q_2 are in coulombs (C); F is in newton (N).

Notes:

- The direction of F is determined using the fact that like charges repel and unlike charges attract.
- r is the distance between the two charges.
- the permittivity of free space $\varepsilon_0 = 8.85 \times 10_{-12}\, F/m$

$$\left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\, Nm^2/C^2 \right)$$

Coulomb's Law

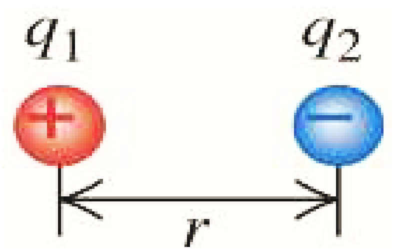


- The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.

Example 1 : Forces between two point charges

Two point charges $q1 = 25\ nC$ and $q_2 = -75\ nC$ are separated by a distance of $3.0\ cm$. Find the magnitude and direction of the electric force that q_1 exerts on $q2$.

Solution:



$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$= 9 \times 10^9\ \frac{(25\times10^{-9})(-75\times10^{-9})}{3\times10^{-2}} = 0.0187\ N$$

The force is attractive

Example 2: **Compare the strength of the electrostatic force between the electron and proton in a hydrogen atom with the corresponding gravitational force between the two. Remember that a hydrogen atom consists of a single electron in orbit around a proton. The electron is pictured as moving around the proton in a circular orbit with radius $r = 5.29*10-11m$. What is the ratio of the magnitude of the electric force between the electron and proton to the magnitude of the gravitational attraction between them?**

$$m_e = 9.1 \times 10^{-31} kg$$
$$m_p = 1.67 \times 10^{-27} kg$$

The gravitational constant is $G = 6.67 \times 10_{-11} Nm_2/kg_2$

Solution: The electric force is given by Coulomb's law and the gravitational force by Newton's law of gravitation.

Each particle has charge of magnitude $e = 1.6 \times 10_{-19} C$.

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = k\frac{e^2}{r^2}$$
$$F_g = G\frac{m_e m_p}{r^2}$$

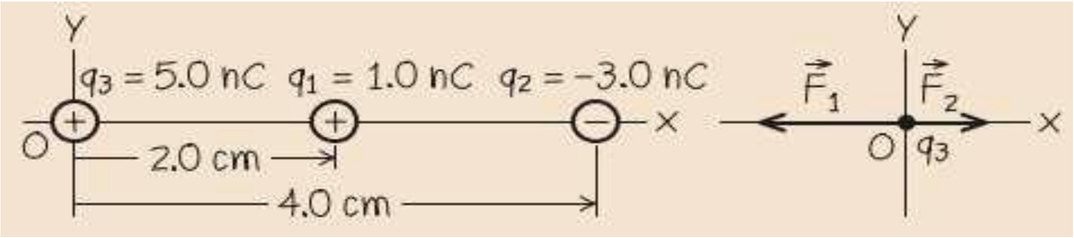
The ratio of the two forces is

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}}{G\frac{m_e m_p}{r^2}} = \frac{ke^2}{Gm_e m_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}}$$
$$= 2.27 \times 10^{39}$$

Example 3:

Vector addition (Superposition) of electric forces on a line

Two point charges are located on the x-axis of a coordinate system: $q_1 = 1.0$ nC is at $x = +2.0$ cm, and $q_2 = -3.0$ nC is at $x = +4.0$ cm. What is the total electric force exerted by q_1 and q_2 on a charge $q_3 = 5.0$ nC at $x = 0$?



Solution:

$$F_{1\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r_{13}^2}$$

$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.020\ \text{m})^2}$$

$$= 1.12 \times 10^{-4}\ \text{N} = 112\ \mu\text{N}$$

In the same way we can show that $\boldsymbol{F}_{2\ on\ 3} = 84\ \mu N$.

Thus we have:

$F_{1\ on\ 3} = -112\ i \quad and\ F_{2\ on\ 3} = 84\ i$

Therefore, the net force on q_3 is

$$F_3 = (-112\mu N)i + (84\mu N)\ i = (-28\mu N)\ i$$

Electric Field

- Definition of the electric field: electric force per unit charge. $\boldsymbol{E} = \boldsymbol{F}/q0$ $\quad the\ SI\ unit\ is\ N/C$ Here,
- q0 is a “test charge” it serves to allow the electric force to be measured, but is not large enough to create a significant force on any other charges.

- If we know the electric field, we can calculate the force on any charge: $\underline{\boldsymbol{F} = q\boldsymbol{E}}$

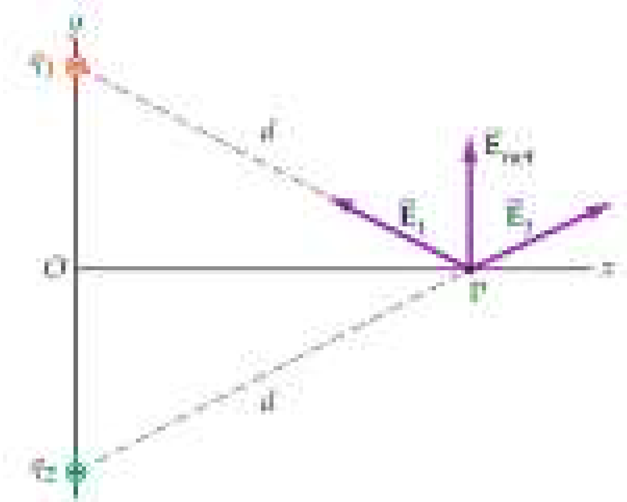
The direction of the force depends on the sign of the charge: in the direction of the field for a positive charge, opposite to it for a negative one.

Superposition principle for electric fields:

Just as electric forces can be superposed, electric fields can as well

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots\dots\dots\dots$

If we place a small test charge q_0 at the field point *P*, at a distance r from the source point, the magnitude of the force is given by Coulomb's law



$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{r^2}$$

the magnitude of the electric field at *P* is

$$\boldsymbol{E} = \frac{\boldsymbol{F_0}}{q_0} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{r^2}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Example 1: What is the magnitude of the electric field at a field point 2.0 m from a point charge q = 4.0 nC

- **Solution**

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{4.0 \times 10^{-9}\ \mathrm{C}}{(2.0\ \mathrm{m})^2}$$

$$= 9.0\ \mathrm{N/C}$$

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Units and Vectors: Tools for Physics

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1.1 The Important Stuff

1.1.1 The SI System

Physics is based on measurement. Measurements are made by comparisons to well–defined standards which define the units for our measurements.

The SI system (popularly known as the *metric system*) is the one used in physics. Its unit of length is the meter, its unit of time is the second and its unit of mass is the kilogram. Other quantities in physics are derived from these. For example the unit of energy is the joule, defined by $1\ \mathrm{J} = 1\ \mathrm{kg\ m^2/s^2}$

As a convenience in using the SI system we can associate prefixes with the basic units to represent powers of 10. The most commonly used prefixes are given here:

Factor	Prefix	Symbol
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	μ
10^{-3}	milli-	m
10^{-2}	centi-	c
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G

1.1.2 Dimensional Analysis

Every equation that we use in physics must have *the same type of units* on both sides of the equals sign. Our basic unit types (dimensions) are length (L), time (T) and mass (M). When we do dimensional analysis we focus on the units of a physics equation without worrying about the numerical values.

1.1.3 Vectors; Vector Addition

Many of the quantities we encounter in physics have both magnitude ("how much") and direction. These are vector quantities.We can represent vectors graphically as arrows and then the sum of two vectors is found (graphically) by joining the head of one to the tail of the other and then connecting head to tail for the combination, as shown in Fig. 1.1 . The sum of two (or more) vectors is often called the resultant.We can add vectors in any order we want: A + B = B + A. We say that vector addition is "commutative".We express vectors in component form using the unit vectors i, j and k, which each have magnitude 1 and point along the x, y and z axes of the coordinate system, respectively.

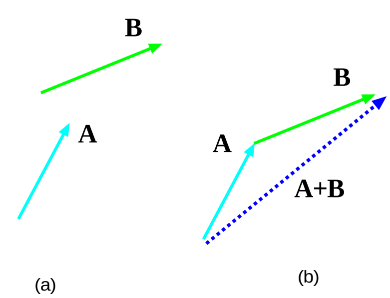


Figure 1.1: Vector addition. (a) shows the vectors A and B to be summed. (b) shows how to perform the sum graphically.

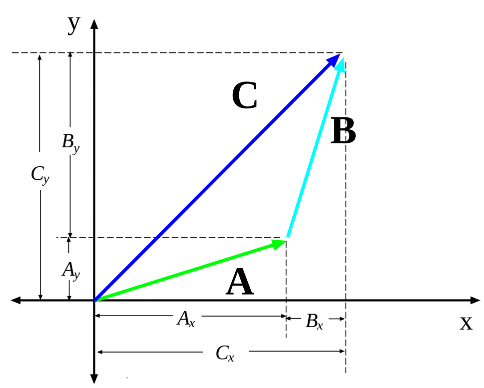


Figure 1.2: Addition of vectors by components (in two dimensions).

Any vector can be expressed as a sum of multiples of these basic vectors; for example, for the vector A we would write:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\ .$$

Here we would say that A_x is the x component of the vector A; likewise for y and z.

In Fig. 1.2 we illustrate how we get the components for a vector which is the *sum* of two other vectors. If

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Then

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k} \tag{1.2}$$

Once we have found the (Cartesian) component of two vectors, addition is simple; just add the *corresponding components* of the two vectors to get the components of the resultant vector.

When we multiply a vector by a scalar, the scalar multiplies each component; If A is a vector and n is a scalar, then

$$c\mathbf{A} = cA_x\mathbf{i} + cA_y\mathbf{j} + cA_z\mathbf{k} \tag{1.3}$$

In terms of its components, the magnitude ("length") of a vector A (which we write as A) is given by:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.4}$$

Many of our physics problems will be in two dimensions (x and y) and then we can also represent it in polar form. If A is a two–dimensional vector and θ as the angle that A makes with the $+x$ axis *measured counter-clockwise* then we can express this vector in terms of components A_x and A_y or in terms of its magnitude A and the angle θ. These descriptions are related by:

$$A_x = A\cos\theta \qquad A_y = A\sin\theta \tag{1.5}$$

$$A = \sqrt{A_x^2 + A_y^2} \qquad \tan\theta = \frac{A_y}{A_x} \tag{1.6}$$

When we use Eq. 1.6 to find θ from A_x and A_y we need to be careful because the inverse tangent operation (as done on a calculator) might give an angle in the wrong quadrant; one must think about the signs of A_x and A_y.

1.1.4 Multiplying Vectors

There are two ways to “multiply” two vectors together.

The scalar product (or dot product) of the vectors a and b is given by

$$\mathrm{a} \cdot \mathrm{b} = ab \cos \varphi \tag{1.7}$$

where a is the magnitude of a, b is the magnitude of b and φ is the angle between a and b.

The scalar product is commutative: a · b = b · a. One can show that a · b is related to the components of a and b by:

$$\mathrm{a} \cdot \mathrm{b} = a_x b_x + a_y b_y + a_z b_z \tag{1.8}$$

If two vectors are perpendicular then their scalar product is *zero*.

The vector product (or cross product) of vectors a and b is a vector c whose magnitude is given by

$$c = ab \sin \varphi \tag{1.9}$$

where φ is the *smallest* angle between a and b. The direction of c is perpendicular to the plane containing a and b with its orientation given by the right–hand rule. One way of using the right–hand rule is to let the fingers of the right hand bend (in their natural direction!) from a to b; the direction of the thumb is the direction of c = a × b. This is illustrated in Fig. 1.3.

The vector product is *anti*–commutative: a × b = b × a.

Relations among the unit vectors for vector products are:

$$\mathrm{i} \times \mathrm{j} = \mathrm{k} \qquad \mathrm{j} \times \mathrm{k} = \mathrm{i} \qquad \mathrm{k} \times \mathrm{i} = \mathrm{j} \tag{1.10}$$

1.2. WORKED EXAMPLES

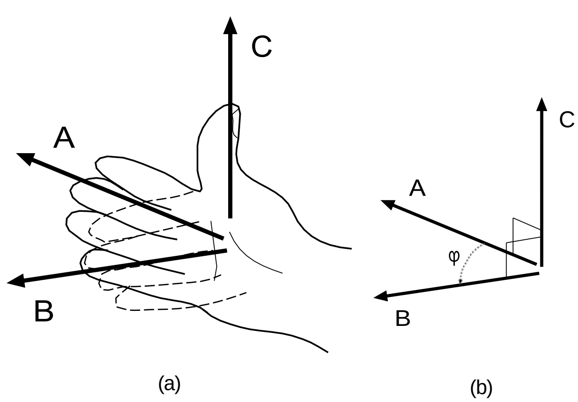


Figure 1.3: (a) Finding the direction of A × B. fingers of the right hand sweep from A to B in the shortest and least painful way. The extended thumb points in the direction of C. (b) Vectors A, B and C. The magnitude of C is $C = AB \sin \varphi$.

The vector product of a and b can be computed from the components of these vectors by:

$$\mathrm{a} \times \mathrm{b} = (a_y b_z - a_z b_y)\mathrm{i} + (a_z b_x - a_x b_z)\mathrm{j} + (a_x b_y - a_y b_x)\mathrm{k} \quad (1.11)$$

which can be abbreviated by the notation of the determinant:

$$\mathrm{a} \times \mathrm{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad (1.12)$$

1.1.5 Vectors; Vector Addition

---

3. (a) What is the sum in unit–vector notation of the two vectors a = 4.0i + 3.0j and b = −13.0i + 7.0j? (b) What are the magnitude and direction of a + b? [HRW5 3-20]

(a) Summing the corresponding components of vectors a and b we find:

$$\begin{aligned} \mathrm{a} + \mathrm{b} &= (4.0 - 13.0)\mathrm{i} + (3.0 + 7.0)\mathrm{j} \\ &= -9.0\mathrm{i} + 10.0\mathrm{j} \end{aligned}$$

This is the sum of the two vectors is unit–vector form.

(b) Using our results from (a), the magnitude of a + b is

$$|\mathrm{a} + \mathrm{b}| = \sqrt{(-9.0)^2 + (10.0)^2} = 13.4$$

and if c = a + b points in a direction ϑ as measured from the positive x axis, then the tangent of ϑ is found from

$$\tan\vartheta = \frac{c_y}{c_x} = -1.11$$

If we naively take the arctangent using a calculator, we are told:

$$\vartheta = \tan^{-1}(-1.11) = -48.0^\circ$$

which is not correct because (as shown in Fig. 1.5), with c_x negative, and c_y positive, the

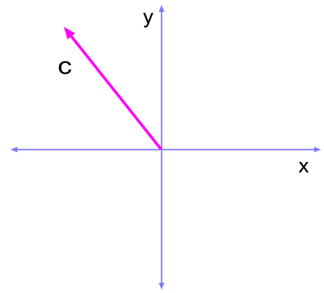


Figure 1.5: Vector c, found in Example 17. With $c_x = -9.0$ and $c_y = +10.0$, the direction of c is in the second quadrant.

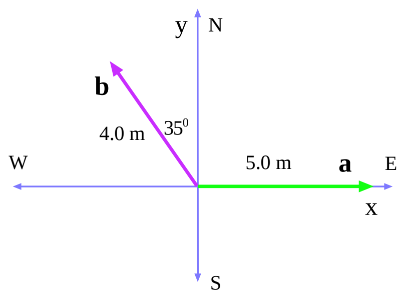


Figure 1.6: Vectors a and b as given in Example 18.

correct angle must be in the *second* quadrant. The calculator was fooled because angles which differ by multiples of 180° have the same tangent. The direction we really want is

$$\vartheta = -48.0^\circ + 180.0^\circ = 132.0^\circ$$

---

4. Vector a has magnitude 5.0 m and is directed east. Vector b has magnitude 4.0 m and is directed 35° west of north. What are (a) the magnitude and (b) the direction of a + b? What are (c) the magnitude and (d) the direction of b − a? Draw a vector diagram for each combination. [HRW6 3-15]

(a) The vectors are shown in Fig. 1.6. (On the axes are shown the common directions N, S, E, W and also the *x* and *y* axes; "North" is the positive *y* direction, "East" is the positive *x* direction, etc.) Expressing the vectors in i, j notation, we have:

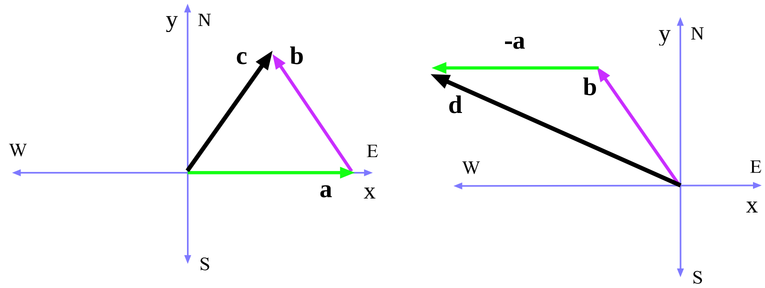
$$\mathrm{a} = (5.00\,\mathrm{m})\mathrm{i}$$

and

$$\begin{aligned} \mathrm{b} &= -(4.00\,\mathrm{m})\sin 35^\circ + (4.00\,\mathrm{m})\cos 35^{c}\, irc \\ &= (-2.29\,\mathrm{m})\mathrm{i} + (3.28\,\mathrm{m})\mathrm{j} \end{aligned}$$

So if vector c is the sum of vectors a and b then:

$$c_x = a_x + b_x = (5.00\,\mathrm{m}) + (-2.29\,\mathrm{m}) = 2.71\,\mathrm{m}$$
$$c_y = a_y + b_y = (0.00\,\mathrm{m}) + (3.28\,\mathrm{m}) = 3.28\,\mathrm{m}$$



(a) (b)

Figure 1.7: (a) Vector diagram showing the addition a + b. (b) Vector diagram showing b − a.

The magnitude of c is

$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71\,\mathrm{m})^2 + (3.28\,\mathrm{m})^2} = 4.25\,\mathrm{m}$$

(b) If the direction of c, as measured counterclockwise from the +*x* axis is ϑ then

$$\tan\vartheta = \frac{c_y}{c_x} = \frac{3.28\,\mathrm{m}}{2.71\,\mathrm{m}} = 1.211$$

then the $\tan^{-1}$ operation on a calculator gives

$$\vartheta = \tan^{-1}(1.211) = 50.4^\circ$$

and since vector c must lie in the first quadrant this angle is correct. We note that this angle is

$$90.0^\circ - 50.4^\circ = 39.6^\circ$$

just shy of the +*y* axis (the "North" direction). So we can also express the direction by saying it is "39.6° East of North".

A vector diagram showing a, b and c is given in Fig. 1.7(a).

(c) If the vector d is given by d = b − a then the components of d are given by

$$d_x = b_x - a_x = (-2.29\,\mathrm{m}) - (5.00\,\mathrm{m}) = -7.29\,\mathrm{m}$$

$$c_y = a_y + b_y = (3.28\,\mathrm{m}) - (0.00\,\mathrm{m}) + (3.28\,\mathrm{m}) = 3.28\,\mathrm{m}$$

The magnitude of c is

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(-7.29\,\mathrm{m})^2 + (3.28\,\mathrm{m})^2} = 8.00\,\mathrm{m}$$

(d) If the direction of d, as measured counterclockwise from the +*x* axis is ϑ then

$$\tan\vartheta = \frac{d_y}{d_x} = \frac{3.28\,\mathrm{m}}{-7.29\,\mathrm{m}} = -0.450$$

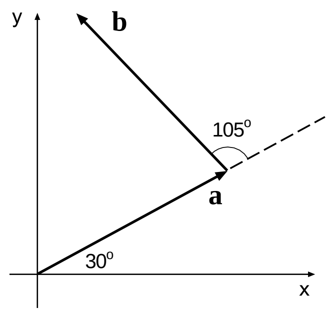


Figure 1.8: Vectors for Example 19.

Naively pushing buttons on the calculator gives

$$\vartheta = \tan^{-1}(-0.450) = -24.2^\circ$$

which can't be right because from the signs of its components we know that d must lie in the second quadrant. We need to add 180° to get the correct answer for the $\tan^{-1}$ operation:

$$\vartheta = -24.2^\circ + 180.0^\circ = 156^\circ$$

But we note that this angle is

$$180^\circ - 156^\circ = 24^\circ$$

shy of the $-y$ axis, so the direction can also be expressed as "24° North of West".

A vector diagram showing a, b and d is given in Fig. 1.7(b).

---

5. The two vectors a and b in Fig. 1.8 have equal magnitudes of 10.0 m. Find (a) the x component and (b) the y component of their vector sum r, (c) the magnitude of r and (d) the angle r makes with the positive direction of the x axis. [HRW6 3-21]

(a) First, find the x and y components of the vectors a and b. The vector a makes an angle of 30° with the $+x$ axis, so its components are

$$\begin{aligned} a_x &= a\cos 30^\circ = (10.0\,\mathrm{m})\cos 30^\circ = 8.66\,\mathrm{m} \\ a_y &= a\sin 30^\circ = (10.0\,\mathrm{m})\sin 30^\circ = 5.00\,\mathrm{m} \end{aligned}$$

The vector b makes an angle of 135° with the $+x$ axis (30° plus 105° more) so its components are

$$\begin{aligned} b_x &= b\cos 135^\circ = (10.0\,\mathrm{m})\cos 135^\circ = -7.07\,\mathrm{m} \\ b_y &= b\sin 135^\circ = (10.0\,\mathrm{m})\sin 135^\circ = 7.07\,\mathrm{m} \end{aligned}$$

Then if r = a + b, the x and y components of the vector r are:

$$\begin{aligned} r_x &= a_x + b_x = 8.66\,\mathrm{m} - 7.07\,\mathrm{m} = 1.59\,\mathrm{m} \\ r_y &= a_y + b_y = 5.00\,\mathrm{m} + 7.07\,\mathrm{m} = 12.07\,\mathrm{m} \end{aligned}$$

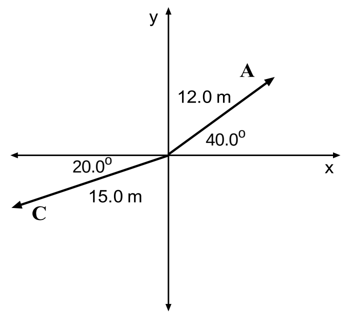


Figure 1.9: Vectors A and C as described in Example 20.

So the x component of the sum is $r_x = 1.59\,\mathrm{m}$, and. . .

(b) . . . the y component of the sum is $r_y = 12.07\,\mathrm{m}$.

(c) The magnitude of the vector r is

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.59\,\mathrm{m})^2 + (12.07\,\mathrm{m})^2} = 12.18\,\mathrm{m}$$

(d) To get the direction of the vector r expressed as an angle ϑ measured from the $+x$ axis, we note:

$$\tan\vartheta = \frac{r_y}{r_x} = 7.59$$

and then take the inverse tangent of 7.59:

$$\vartheta = \tan^{-1}(7.59) = 82.5^\circ$$

Since the components of r are both positive, the vector *does* lie in the first quadrant so that the inverse tangent operation has (this time) given the correct answer. So the direction of r is given by $\vartheta = 82.5^\circ$.

---

6. In the sum A + B = C, vector A has a magnitude of 12.0 m and is angled 40.0° counterclockwise from the $+x$ direction, and vector C has magnitude of 15.0 m and is angled 20.0° counterclockwise from the $-x$ direction. What are (a) the magnitude and (b) the angle (relative to $+x$) of B? [HRW6 3-22]

(a) Vectors A and C are diagrammed in Fig. 1.9. From these we can get the components of A and C (watch the signs on vector C from the odd way that its angle is given!):

$$A_x = (12.0\,\mathrm{m})\cos(40.0^\circ) = 9.19\,\mathrm{m} \qquad A_y = (12.0\,\mathrm{m})\sin(40.0^\circ) = 7.71\,\mathrm{m}$$

$$C_x = -(15.0\,\mathrm{m})\cos(20.0^\circ) = -14.1\,\mathrm{m} \qquad C_y = -(15.0\,\mathrm{m})\sin(20.0^\circ) = -5.13\,\mathrm{m}$$

(Note, the vectors in this problem have *units* to go along with their magnitudes, namely m (meters).) Then from the relation A + B = C it follows that B = C − A, and from this we find the components of B:

$$B_x = C_x - A_x = -14.1\,\mathrm{m} - 9.19\,\mathrm{m} = -23.3\,\mathrm{m}$$

$$B_y = C_y - A_y = -5.13\,\text{m} - 7.71\,\text{m} = -12.8\,\text{m}$$

Then we find the magnitude of vector B:

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-23.3)^2 + (-12.8)^2}\,\text{m} = 26.6\,\text{m}$$

(b) We find the direction of B from:

$$\tan\vartheta = \frac{B_y}{B_x} = 0.551$$

If we *naively* press the "atan" button on our calculators to get ϑ, we are told:

$$\vartheta = \tan^{-1}(0.551) = 28.9^\circ \qquad (?)$$

which cannot be correct because from the components of B (both negative) we know that vector B lies in the third quadrant. So we need to ad 180° to the naive result to get the *correct* answer:

$$\vartheta = 28.9^\circ + 180.0^\circ = 208.9^\circ\,.$$

This is the angle of B, measured counterclockwise from the $+x$ axis.

---

7. If a − b = 2c, a + b = 4c and c = 3i + 4j, then what are a and b? [HRW5 3-24]

We notice that if we add the first two relations together, the vector b will cancel:

$$(a - b) + (a + b) = (2c) + (4c)$$

which gives:

$$2a = 6c \qquad \Longrightarrow \qquad a = 3c$$

and we can use the last of the given equations to substitute for c; we get

$$a = 3c = 3(3i + 4j) = 9i + 12j$$

Then we can rearrange the first of the equations to solve for b:

$$\begin{aligned} b &= a - 2c = (9i + 12j) - 2(3i + 4j) \\ &= (9 - 6)i + (12 - 8)j \\ &= 3i + 4j \end{aligned}$$

So we have found:

$$a = 9i + 12j \qquad \text{and} \qquad b = 3i + 4j$$

---

8. If A = (6.0i − 8.0j) units, B = (−8.0i + 3.0j) units, and C = (26.0i + 19.0j) units, determine a and b so that aA + bB + C = 0. [Ser4 3-46]

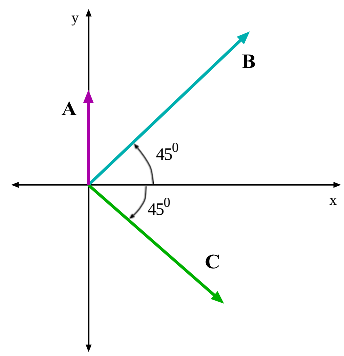


Figure 1.10: Vectors for Example 23

The condition on the vectors given in the problem:

$$a\mathbf{A} + b\mathbf{B} + \mathbf{C} = 0$$

is a condition on the *individual components* of the vectors. It implies:

$$aA_x + bB_x + C_x = 0 \qquad \text{and} \qquad aA_y + bB_y + C_y = 0 \ .$$

So that we have the equations:

$$\begin{aligned} 6.0a - 8.0b + 26.0 &= \quad 0 \\ -8.0a + 3.0b + 19.0 &= \quad = 0 \end{aligned}$$

We have two equations for two unknowns so we *can* find a and b. The are lots of ways to do this; one could multiply the first equation by 4 and the second equation by 3 to get:

$$\begin{aligned} 24.0a - 32.0b + 104.0 &= \quad 0 \\ -24.0a + 9.0b + 57.0 &= \ = 0 \end{aligned}$$

Adding these gives

$$\_\_23.0b + 161 = 0 \qquad \Longrightarrow \qquad b = \frac{-161.0}{-23.0} = 7.0$$

and then the first of the original equations gives us a:

$$6.0a = 8.0b - 26.0 = 8.0(7.0) - 26.0 = 30.0 \qquad \Longrightarrow \qquad a = \frac{30.0}{6.0} = 5.0$$

and our solution is

$$a = 7.0 \qquad\qquad b = 5.0$$

---

9. Three vectors are oriented as shown in Fig. 1.10, where $|\mathbf{A}| = 20.0$ units, $|\mathbf{B}| = 40.0$ units, and $|\mathbf{C}| = 30.0$ units. Find (a) the x and y components of the

resultant vector and (b) the magnitude and direction of the resultant vector. [Ser4 3-47]

(a) Let's first put these vectors into "unit–vector notation":

$$\begin{aligned}
A &= 20.0j \\
B &= (40.0\cos 45^\circ)i + (40.0\sin 45^\circ)j = 28.3i + 28.3j \\
C &= (30.0\cos(-45^\circ))i + (30.0\sin(-45^\circ))j = 21.2i - 21.2j
\end{aligned}$$

Adding the components together, the resultant (total) vector is:

$$\begin{aligned}
\text{Resultant} &= A + B + C \\
&= (28.3 + 21.2)i + (20.0 + 28.3 - 21.2)j \\
&= 49.5i + 27.1j
\end{aligned}$$

So the x component of the resultant vector is 49.5 and the y component of the resultant is 27.1.

(b) If we call the resultant vector R, then the magnitude of R is given by

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(49.5)^2 + (27.1)^2} = 56.4$$

To find its direction (given by ϑ, measured counterclockwise from the x axis), we find:

$$\tan\vartheta = \frac{R_y}{R_x} = \frac{27.1}{49.5} = 0.547$$

and then taking the inverse tangent gives a *possible* answer for ϑ:

$$\vartheta = \tan^{-1}(0.547) = 28.7^\circ .$$

Is this the right answer for ϑ? Since *both* components of R are *positive*, it must lie in the first quadrant and so ϑ must be between 0° and 90°. So the direction of R is given by 28.7°.

---

10. A vector B, when added to the vector C = 3.0i + 4.0j, yields a resultant vector that is in the positive y direction and has a magnitude equal to that of C. What is the magnitude of B? [HRW5 3-26]

If the vector B is denoted by $B = B_x i + B_y j$ then the resultant of B and C is

$$B + C = (B_x + 3.0)i + (B_y + 4.0)j .$$

We are told that the resultant points in the positive y direction, so its x component must be *zero*. Then:

$$B_x + 3.0 = 0 \qquad \Longrightarrow \qquad B_x = -3.0 .$$

Now, the magnitude of C is

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(3.0)^2 + (4.0)^2} = 5.0$$

so that if the magnitude of B + C is also 5.0 then we get

$$|\mathbf{B} + \mathbf{C}| = \sqrt{(0)^2 + (B_y + 4.0)^2} = 5.0 \qquad \Longrightarrow \qquad (B_y + 4.0)^2 = 25.0\ .$$

The last equation gives $(B_y + 4.0) = \pm 5.0$ and apparently there are *two* possible answers

$$B_y = +1.0 \qquad \text{and} \qquad B_y = -9.0$$

but the second case gives a resultant vector B + C which points in the *negative* y direction so we omit it. Then with $B_y = 1.0$ we find the magnitude of B:

$$B = \sqrt{(B_x)^2 + (B_y)^2} = \sqrt{(-3.0)^2 + (1.0)^2} = 3.2$$

The magnitude of vector B is 3.2.

1.1.6 Multiplying Vectors

---

11. Vector A extends from the origin to a point having polar coordinates $(7, 70^\circ)$ and vector B extends from the origin to a point having polar coordinates $(4, 130^\circ)$. Find $\mathbf{A}\cdot\mathbf{B}$. [Ser4 7-13]

We can use Eq. 1.7 to find $\mathbf{A}\cdot\mathbf{B}$. We have the magnitudes of the two vectors (namely $A = 7$ and $B = 4$) and the angle φ between the two is

$$\varphi = 130^\circ - 70^\circ = 60^\circ\ .$$

Then we get:

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\varphi = (7)(4)\cos 60^\circ = 14$$

---

12. Find the angle between $\mathbf{A} = -5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = -2\mathbf{j} - 2\mathbf{k}$.

Eq. 1.7 allows us to find the cosine of the angle between two vectors as long as we know their magnitudes and their dot product. The magnitudes of the vectors A and B are:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-5)^2 + (-3)^2 + (2)^2} = 6.164$$
$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(0)^2 + (-2)^2 + (-2)^2} = 2.828$$

and their dot product is:

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z = (-5)(0) + (-3)(-2) + (2)(-2) = 2$$

Then from Eq. 1.7, if φ is the angle between A and B, we have

$$\cos\varphi = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{2}{(6.164)(2.828)} = 0.114$$

which then gives

$$\varphi = 83.4^\circ .$$

---

13. Two vectors a and b have the components, in arbitrary units, $a_x = 3.2$, $a_y = 1.6$, $b_x = 0.50$, $b_y = 4.5$. (a) Find the angle between the directions of a and b. (b) Find the components of a vector c that is perpendicular to a, is in the xy plane and has a magnitude of 5.0 units. [HRW5 3-51]

(a) The scalar product has something to do with the angle between two vectors... if the angle between a and b is φ then from Eq. 1.7 we have:

$$\cos\varphi = \frac{\mathbf{a}\cdot\mathbf{b}}{ab} .$$

We can compute the right–hand–side of this equation since we know the components of a and b. First, find a · b. Using Eq. 1.8 we find:

$$\begin{aligned} \mathbf{a}\cdot\mathbf{b} &= a_xb_x + a_yb_y \\ &= (3.2)(0.50) + (1.6)(4.5) \\ &= 8.8 \end{aligned}$$

Now find the magnitudes of a and b:

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(3.2)^2 + (1.6)^2} = 3.6$$
$$b = \sqrt{b_x^2 + b_y^2} = \sqrt{(0.50)^2 + (4.5)^2} = 4.5$$

This gives us:

$$\cos\varphi = \frac{\mathbf{a}\cdot\mathbf{b}}{ab} = \frac{8.8}{(3.6)(4.5)} = 0.54$$

From which we get φ by:

$$\varphi = \cos^{-1}(0.54) = 57^\circ$$

(b) Let the components of the vector c be c_x and c_y (we are told that it lies in the xy plane). If c is perpendicular to a then the dot product of the two vectors must give zero. This tells us:

$$\mathbf{a}\cdot\mathbf{c} = a_xc_x + a_yc_y = (3.2)c_x + (1.6)c_y = 0$$

This equation doesn't allow us to solve for the components of c but it does give us:

$$c_x = -\frac{1.6}{3.2}c_y = -0.50c_y$$

Since the vector c has magnitude 5.0, we know that

$$c = \sqrt{c_x^2 + c_y^2} = 5.0$$

Using the previous equation to substitute for c_x gives:

$$\begin{aligned} c &= \sqrt{c_x^2 + c_y^2} \\ &= \sqrt{(-0.50\, c_y)^2 + c_y^2} \\ &= \sqrt{1.25\, c_y^2} = 5.0 \end{aligned}$$

Squaring the last line gives

$$1.25 c_y^2 = 25 \qquad \Longrightarrow \qquad c_y^2 = 20. \qquad \Longrightarrow \qquad c_y = \pm 4.5$$

One must be careful... there are *two* possible solutions for c_y here. If $c_y = 4.5$ then we have

$$c_x = -0.50\, c_y = (-0.50)(4.5) = -2.2$$

But if $c_y = -4.5$ then we have

$$c_x = -0.50\, c_y = (-0.50)(-4.5) = 2.2$$

So the two possibilities for the vector c are

$$c_x = -2.2 \qquad c_y = 4.5$$

and

$$c_x = 2.2 \qquad c_y = -4.5$$

---

14. Two vectors are given by A = −3i + 4j and B = 2i + 3j. Find (a) A × B and (b) the angle between A and B. [Ser4 11-7]

(a) Setting up the determinant in Eq. 1.12 (or just using Eq. 1.11 for the cross product) we find:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = (0-0)\mathbf{i} + (0-0)\mathbf{j} + ((-9)-(8))\mathbf{k} = -17\mathbf{k}$$

(b) To get the angle between A and B it is easiest to use the dot product and Eq. 1.7. The magnitudes of A and B are:

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3)^2 + (4)^2} = 5 \quad B = \sqrt{B_x^2 + B_y^2} = \sqrt{(2)^2 + (3)^2} = 3.61$$

and the dot product of the two vectors is

$$\mathrm{A} \cdot \mathrm{B} = A_x B_x + A_y B_y + A_z B_z = (-3)(2) + (4)(3) = 6$$

so then if φ is the angle between A and B we get:

$$\cos \varphi = \frac{\mathrm{A} \cdot \mathrm{B}}{AB} = \frac{6}{(5)(3.61)} = 0.333$$

which gives

$$\varphi = 70.6^\circ .$$

---

15. Prove that two vectors must have equal magnitudes if their sum is perpendicular to their difference. [HRW6 3-23]

Suppose the condition stated in this problem holds for the two vectors a and b. If the sum a + b is perpendicular to the difference a − b then the dot product of these two vectors is zero:

$$(\mathrm{a} + \mathrm{b}) \cdot (\mathrm{a} - \mathrm{b}) = 0$$

Use the distributive property of the dot product to expand the left side of this equation. We get:

$$\mathrm{a} \cdot \mathrm{a} - \mathrm{a} \cdot \mathrm{b} + \mathrm{b} \cdot \mathrm{a} - \mathrm{b} \cdot \mathrm{b}$$

But the dot product of a vector with itself gives the magnitude squared:

$$\mathrm{a} \cdot \mathrm{a} = a_x^2 + a_y^2 + a_z^2 = a^2$$

(likewise b· b = b^2) and the dot product is commutative: a · b = b · a. Using these facts, we then have

$$a^2 - \mathrm{a} \cdot \mathrm{b} + \mathrm{a} \cdot \mathrm{b} + b^2 = 0 \ ,$$

which gives:

$$a^2 - b^2 = 0 \qquad \Longrightarrow \qquad a^2 = b^2$$

Since the magnitude of a vector must be a positive number, this implies $a = b$ and so vectors a and b have the same magnitude.

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16. For the following three vectors, what is 3C · (2A × B) ?

$$\mathrm{A} = 2.00\mathrm{i} + 3.00\mathrm{j} - 4.00\mathrm{k}$$

$$\mathrm{B} = -3.00\mathrm{i} + 4.00\mathrm{j} + 2.00\mathrm{k} \qquad \mathrm{C} = 7.00\mathrm{i} - 8.00\mathrm{j}$$

Actually, from the properties of scalar multiplication we can combine the factors in the desired vector product to give:

$$3\mathbf{C} \cdot (2\mathbf{A} \times \mathbf{B}) = 6\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \ .$$

Evaluate $\mathbf{A} \times \mathbf{B}$ first:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.0 & 3.0 & -4.0 \\ -3.0 & 4.0 & 2.0 \end{vmatrix} = (6.0 + 16.0)\mathbf{i} + (12.0 - 4.0)\mathbf{j} + (8.0 + 9.0)\mathbf{k}$$

$$= 22.0\mathbf{i} + 8.0\mathbf{j} + 17.0\mathbf{k}$$

Then:

$$\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = (7.0)(22.0) - (8.0)(8.0) + (0.0)(17.0) = 90$$

So the answer we want is:

$$6\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = (6)(90.0) = 540$$

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Field of an electric dipole

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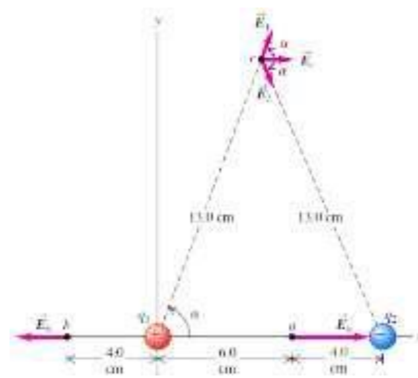
Field of an electric dipole

Example3: Point charges $q1$ and $q2$ are 0.1 m apart. (Such pairs of point charges with equal magnitude and opposite sign are called electric dipoles.) Compute the electric field caused by $q1$,the field caused by $q2$ and the total field (a) at point a (b) at point b, and (c) at point c

Solution:

We must find the total electric field at various points due to two point charges. We use the principle of superposition: $\boldsymbol{E} = \boldsymbol{E1} + \boldsymbol{E2}$**.** The field points a, b. and c are shown in the figure.

EXECUTE: At each field point, E depends on $\boldsymbol{E1}$ and $\boldsymbol{E2}$there; we first calculate the magnitudes $\boldsymbol{E1}$and $\boldsymbol{E2}$ at each field point. At a the magnitude of the field $E1a$ caused by q_1 is



We calculate the other field magnitudes in a similar way. The results are

$$E_{1a} = 3.0 \times 10^4 \text{ N/C} \qquad E_{1b} = 6.8 \times 10^4 \text{ N/C}$$
$$E_{1c} = 6.39 \times 10^3 \text{ N/C}$$
$$E_{2a} = 6.8 \times 10^4 \text{ N/C} \qquad E_{2b} = 0.55 \times 10^4 \text{ N/C}$$
$$E_{2c} = E_{1c} = 6.39 \times 10^3 \text{ N/C}$$

The *directions* of the corresponding fields are in all cases *away* from the positive charge q_1 and *toward* the negative charge q_2.

the directions of and at c. Both vectors have the same x-component:

$$E_{1cx} = E_{2cx} = E_{1c}\cos\alpha = (6.39 \times 10^3 \text{ N/C})\left(\tfrac{5}{13}\right)$$
$$= 2.46 \times 10^3 \text{ N/C}$$

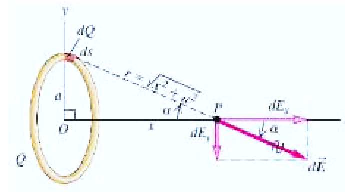
From symmetry, E_{1y} and E_{2y} are equal and opposite, so their sum is zero. Hence

$$\vec{E}_c = 2(2.46 \times 10^3 \text{ N/C})\hat{\imath} = (4.9 \times 10^3 \text{ N/C})\hat{\imath}$$

Field of a ring of charge

Example 4: Charge is uniformly distributed around a conducting ring of radius. Find the electric field at a point *P* on the ring axis at a distance *x* from its center.

Solution: To calculate , divide the ring into small segments ds, so the electric field at P due to the segment ds is



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}$$

The x-component of this field is

$$dE_x = dE\cos\alpha.$$

The charge on the segment ds is

$$dQ = \lambda\, ds,$$

where λ is the linear charge density

$$\lambda = Q/2\pi a$$

$$r^2 = x^2 + a^2$$

$$\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$$

$$\therefore dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\lambda\, ds}{(x^2+a^2)^{3/2}}$$

To find E_x we integrate this expression over the entire ring circumference that is, for s from 0 to $2\pi a$.

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+a^2)^{3/2}}\int_0^{2\pi a} ds$$

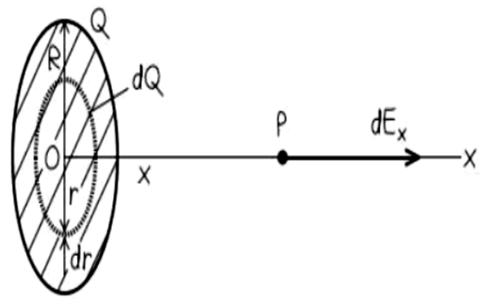
$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+a^2)^{\frac{3}{2}}}(2\pi a) = \frac{1}{4\pi\varepsilon_0}\frac{x(\frac{Q}{2\pi a})}{(x^2+a^2)^{3/2}}(2\pi a)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+a^2)^{\frac{3}{2}}} \qquad \text{in the } +x - direction.$$

Field of a uniformly charged disk

Example 5: A non-conducting disk of radius R has a uniform positive surface charge density . Find the electric field at a point along the axis of the disk a distance x from its center. Assume that x is positive.

Solution: the disk is a set of concentric rings. A typical ring has a charge , inner radius r, and outer radius $r + dr$.



$$dA = 2\pi r dr$$

The charge per unit surface area is $\sigma = \frac{dQ}{dA}$, so the charge of the ring is

$$dQ = \sigma\, dA = 2\pi\sigma r dr$$

The field component dE_x at point P due to this ring (Similar to example 4 and replacing the ring radius a with r.) is

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2 + a^2}\frac{x}{\sqrt{x^2 + r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\sigma\, dA}{(x^2 + r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r, from $r = 0\ to\ r = R$

$$E_x = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{x(2\pi\sigma r dr)}{(x^2 + r^2)^{3/2}} = \frac{\sigma x}{4\varepsilon_0}\int_0^R \frac{2r dr}{(x^2 + r^2)^{3/2}}$$

Let $t = x^2 + r^2$, so $dt = 2r dr$, the result is

$$E_x = \frac{\sigma x}{4\varepsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2) + 1}}\right]$$

Note that if the disk is very large (or we are very close to it), so that $R \gg x$, the term $\frac{1}{\sqrt{(R^2/x^2)+1}}$ will be much less than 1. then the field becomes

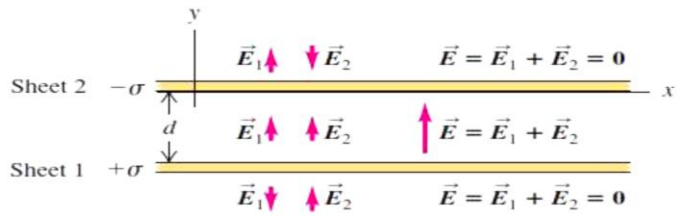
$$E = \frac{\sigma}{2\varepsilon_0}$$

This result shows that for an infinite plane sheet of charge the field is independent of the distance from the sheet.

The direction of the field is perpendicularly away from the sheet.

Field of two oppositely charged infinite sheets

Example 6: Two infinite plane sheets with uniformsurface charge densities and are placed parallel to each other with separation. Find the electric field between the sheets, above the upper sheet, and below the lower sheet.



Solution: both $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ have the same magnitude at all points, independent of distance from either sheet.

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

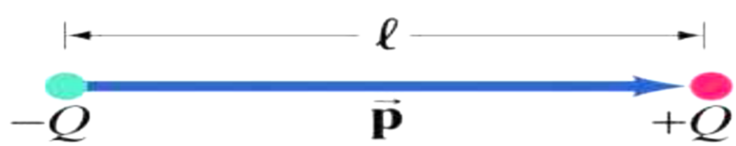
E_1 is everywhere directed away from sheet 1(+ charge), and $E_2$is everywhere directed toward sheet 2 (- charge).

Between the sheets, $E_1$and E_2 reinforce each other; above the upper sheet and below the lower sheet, they cancel each other. Thus the total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \dfrac{\sigma}{\epsilon_0}\hat{j} & \text{between the sheets} \\ 0 & \text{below the lower sheet} \end{cases}$$

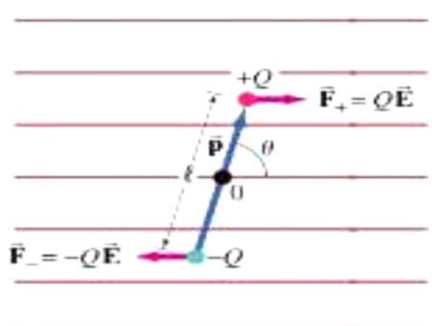
Electric Dipoles

An electric dipole consists of two charges Q, equal in magnitude and opposite in sign, separated by a distance .



The dipole moment, $\mathbf{p} = Ql$, points from the negative to the positive charge. An electric dipole in a uniform electric field will experience no net force, but it will, in general, experience a torque:

$$\boxed{\tau = QE\frac{\ell}{2}\sin\theta + QE\frac{\ell}{2}\sin\theta = pE\sin\theta.}$$



$$\vec{\tau} = \vec{p} \times \vec{E}.$$

• The torque is maximum when and $\boldsymbol{p}$ and $\boldsymbol{E}$ are perpendicular and is zero when they are parallel or antiparallel.

➢ The torque always tends to turn $\boldsymbol{p}$ to line up with $\boldsymbol{E}$.

➢ The position of stable equilibrium occurs when $\varphi = 0$ (p and E are parallel) and when $\varphi = \pi$ (p and E are antiparallel) is a position of unstable equilibrium.

Potential Energy of an Electric Dipole

When a dipole changes direction in an electric field, the electric-field torque does work on it, with a corresponding change in potential energy.

The work done by a torque during an infinitesimal displacement is $d\theta$ is given by

$$dW = \tau\, d\theta = -pE\sin\theta$$

displacement from θ_1 to θ_2 the total work done on the dipole is

$$W = \int_{\theta 1}^{\theta 2} (-pE\sin\theta)\, d\theta$$

$$W = pE\cos\theta_2 - pE\cos\theta_1$$

The work is the negative of the change of potential energy

$$W = U1 - U2$$

So a suitable definition of potential energy for this system is

$$U(\theta) = -pE\cos\theta$$

Since $pE\cos\theta = p \cdot E$ (scalar product)

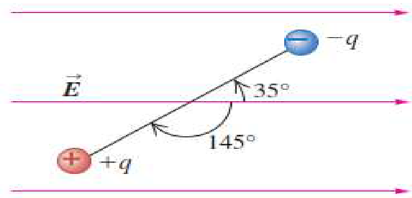
$$\therefore \boldsymbol{U = -p \cdot E}$$

➢ The potential energy has its minimum (most negative) value $U = -pE$ at the stable equilibrium position, where $\theta = 0$ and $\boldsymbol{p}$ is parallel to $\boldsymbol{E}$

➢ The potential energy is maximum when $\theta = \pi$ and p is antiparallel to E then $U = +pE$

➢ A $\theta = \pi/2$ t where $\boldsymbol{p}$ is perpendicular to $\boldsymbol{E}$, $\boldsymbol{U = 0}$

Example: the figure shows an electric dipole in a uniform electric field of magnitude $5 \times 10^5\ N/C$ that is directed parallel to the plane of the figure. The charges are $\mp 1.6 \times 10^{-19} C$; both lie in the plane and are separated by $0.125\ mm$. Find:



(a) The net force exerted by the field on the dipole.
(b) The magnitude and direction of the dipole moment.
(c) The magnitude and direction of the torque.
(d) The potential energy of the system in the position shown.

Solution:

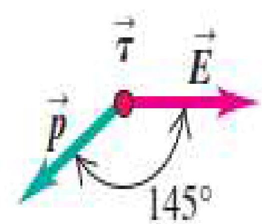
(a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b)

(b) the magnitude of the electric dipole moment is

$$p = qd = 1.6 \times 10^{-19} \times 0.125 \times 10^{-3} = 2 \times 10^{-29} C.m$$

The direction of $\boldsymbol{p}$ is from negative to positive charge, 145^0 clockwise from the direction of the electric field.



(c) The magnitude of the torque is

$$\tau = pE\sin\theta = 2 \times 10^{-29} \times 5 \times 10^{5} \times \sin 145^{0}$$

$$= 5.7 \times 10^{-24} N.m$$

The direction of the torque $\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E}$ is out of the page (right hand rule).this corresponds to a counterclockwise torque that tends to align $\boldsymbol{p}$ with $\boldsymbol{E}$.

(c) The potential energy is

$$U = -pE\cos\theta = -2 \times 10^{-29} \times 5 \times 10^{5} \times \cos 145^{0} = 8.2 \times 10^{-24} J$$